040225E 東角 比呂志

[1] C を積分定数とする.

(1) $\int \frac{dx}{x^2+x} = \int \left(\frac{1}{x-1} - \frac{1}{x}\right) dx = \log\left|\frac{x-1}{x}\right| + C$.

(2) $\int \frac{dx}{\sqrt{x^2+1}}$ $t - x = \sqrt{x^2+1}$ とおくと $\frac{dt}{dx} = \frac{x}{\sqrt{x^2+1}} + 1 = \frac{t}{\sqrt{x^2+1}}$

$= \int \frac{1}{\sqrt{x^2+1}} \times \frac{\sqrt{x^2+1}}{t} dt$

$= \log t + C$

$= \log\left|x + \sqrt{x^2+1}\right| + C$

(3) $\int \sin \log x \, dx = x \sin \log x + \int \cos \log x \, dx$.

$= x \sin \log x + x \cos \log x - \int \sin \log x$

$\therefore \int \sin \log x = \frac{x}{2}(\sin \log x + \cos \log x) + C$.

[2] $y = 4x^2$ の $(0,0)$ から $(a, 4a^2)$ までの長さを $l(a)$ とおくと, $y' = 8x$.

$l(a) = \int_0^a \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx = \int_0^a \sqrt{1+64x^2} dx$

$t - 8x = \sqrt{1+64x^2}$ とおくと $x = \frac{t^2-1}{16t}$

$\frac{dt}{dx} = 8 \times \frac{t}{\sqrt{64x^2+1}}$, $\alpha = \sqrt{64a^2+1} + 8a$ とおくと

$l(a) = \int_1^\alpha \frac{1}{8t} \left\{64 \cdot \frac{(t^2-1)^2}{16^2 t^2} + 1\right\} dt$

$= \int_1^\alpha \frac{1}{8t} \times \frac{(t^2+1)^2}{4t^2} dt$

$= \frac{1}{32} \int_1^\alpha \frac{t^4+2t^2+1}{t^3} dt$

$= \frac{1}{32} \left[\frac{1}{2}t^2 + 2\log t - \frac{1}{2t^2}\right]_1^\alpha$

$= \frac{1}{64}\left(\alpha + \frac{1}{\alpha}\right)\left(\alpha - \frac{1}{\alpha}\right) + \frac{1}{16}\log\left|\sqrt{64a^2+1} + 8a\right|$

$= \frac{1}{2} a \sqrt{64a^2+1} + \frac{1}{16}\log\left|\sqrt{64a^2+1} + 8a\right|$