

$$i) G_x = \frac{2}{9}x, \quad G_y = \frac{1}{2}y$$

$$G_x = G_y = 0 \iff (a, b) = (0, 0) \leftarrow G(x, y) = 0 \text{ の上にある特異点 BL.}$$

$$ii) F(x, y, \lambda) = x^2y^2 - 2xy - \lambda \left(\frac{x^2}{9} + \frac{y^2}{4} - 1 \right)$$

$$\begin{cases} F_x = 0 \\ F_y = 0 \\ G(a, b) = 0 \end{cases} \iff \begin{cases} 2ab^2 - 2b = \frac{2}{9}\lambda a & \text{--- } \textcircled{1} \\ 2a^2b - 2a = \frac{1}{2}\lambda b & \text{--- } \textcircled{2} \\ \frac{a^2}{9} + \frac{b^2}{4} - 1 = 0 & \text{--- } \textcircled{3} \end{cases}$$

$$\text{r.c.c. } (a, b) = \left(\pm \frac{3}{\sqrt{2}}, \pm \sqrt{2} \right), \quad \left(\pm \frac{3}{\sqrt{2}}, \mp \sqrt{2} \right)$$

$$\left(\pm \sqrt{\frac{9+6\sqrt{2}}{2}}, \pm \sqrt{\frac{2}{9+6\sqrt{2}}} \right), \quad \left(\pm \sqrt{\frac{9-6\sqrt{2}}{2}}, \pm \sqrt{\frac{2}{9-6\sqrt{2}}} \right)$$

$f(a, b)$ の値.

$$f(a, b) = \begin{cases} 3 & \textcircled{1} \\ 15 & \textcircled{2} \\ -1 & \textcircled{3} \end{cases} \leftarrow \begin{matrix} \text{最大 } (2, 2) \\ \text{最小 } (4, 2) \end{matrix}$$

$$\textcircled{31) } \begin{cases} x = 3\cos\theta \\ y = 2\sin\theta \end{cases} \rightarrow \frac{x^2}{9} + \frac{y^2}{4} = \cos^2\theta + \sin^2\theta = 1$$

$$f(x, y) = f(3\cos\theta, 2\sin\theta)$$

$$= (8\sin^2\theta) - 6\sin^2\theta \leftarrow \text{最大値 } 2 \text{ (} \theta = \frac{\pi}{2} \text{), 最小値 } -6 \text{ (} \theta = 0 \text{)}$$