Date

この時、陰関数定理から、局所的には $y=\varphi(x)$ とかける。$(\varphi(a)=b)$

$z=f(x,\varphi(x))$

$$\frac{dz}{dx}=\frac{\partial f}{\partial x}\cdot\underbrace{\frac{dx}{dx}}_{=1}+\frac{\partial f}{\partial y}\cdot\frac{dy}{dx}$$

$$\therefore \frac{dz}{dx}(a)=\frac{\partial f}{\partial x}(a,\varphi(a))+\frac{\partial f}{\partial y}(a,\underbrace{\varphi(a)}_{=b})\frac{d\varphi}{dx}(a)$$

陰関数定理(3)

$$=\frac{\partial f}{\partial x}(a,\varphi(a))+\frac{\partial f}{\partial y}(a,\varphi(a))\left(-\frac{\frac{\partial G}{\partial x}(a,\varphi(a))}{\frac{\partial G}{\partial y}(a,\varphi(a))}\right)$$

極値をとる $\Rightarrow$ $\frac{dz}{dx}(a)=0$

$$\therefore \frac{f_x(a,b)}{G_x(a,b)}=\frac{f_y(a,b)}{G_y(a,b)}=\lambda \text{ とおく。}$$

(a,b,λ) は、

$F_x(a,b,\lambda)=f_x(a,b)-G_x(a,b)=0$

$(F=f-\lambda G$

$F_y(a,b,\lambda)=f_y(a,b)-G_y(a,b)=0$

$G(a,b)=0$

もし、$G_x(a,b)=0$ なら、

$$\frac{d\varphi}{dx}(a)=-\frac{G_x(a,b)}{G_y(a,b)}=0$$

$$\therefore \frac{dz}{dx}(a)=\frac{\partial f}{\partial x}(a,b)=0$$

このCaseでも、$\lambda=\frac{f_y(a,b)}{G_y(a,b)}$ とおけば、

$$\begin{cases}F_x(a,b,\lambda)=0\\ F_y(a,b,\lambda)=0\\ G(a,b)=0\end{cases}$$