

$$100! = 100 \cdot 99 \cdot 98 \cdots 1$$

$$100^{100} = 100 \cdot 100 \cdots 100$$

No. \_\_\_\_\_

Date. \_\_\_\_\_

Stirlingの近似  $n! \sim \left(\frac{n}{e}\right)^n \quad n \gg 1$ .

$$n! = n(n-1) \cdots 2 \cdot 1$$

$$n! \sim n^n$$

$$\begin{aligned} n! &\leq n^n \\ \log n! &= \sum_{k=1}^n \log k \approx \int_1^n dx \log x \\ &= [x \log x - x]_1^n \end{aligned}$$

$$= n \log n - n$$

$$= n(\log n - 1)$$

$$= n \log \left(\frac{n}{e}\right) \left(= \log \left(\frac{n}{e}\right)^n\right)$$

$$\int_0^n \log x \, dx \leq \log n! \leq \int_1^{n+1} dx \log x$$

$$\left(\frac{n}{e}\right)^n \leq n! \leq (n+1) \left(\frac{n+1}{e}\right)^n \quad (\text{E5})$$

$$n! \sim \sqrt{2\pi n} \left(\frac{n}{e}\right)^n \quad n \gg 1$$

$$\lim_{n \rightarrow \infty} \frac{n!}{\sqrt{2\pi n} \left(\frac{n}{e}\right)^n} = 1$$

証明は略 (これは本nの213.)