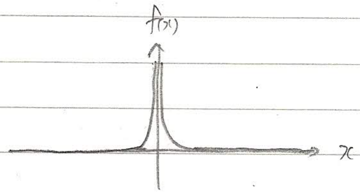
# 第2章 Maxwell 方程式の一般的性質

## §1. 点電荷と電磁場の共存する体系

・点電荷の数学的表現

Dirac の δ-関数 (超関数)

$$\delta(x-a) = \begin{cases} \infty & (x=a) \\ 0 & (x \neq a) \end{cases}$$




$$\int_{-\infty}^{\infty} \delta(x)\, dx = 1$$

グラフより. $\delta(x) = \delta(-x)$ , よって

$$\int_{-\infty}^{\infty} \delta(x-a) f(x)\, dx = f(a)$$

・電荷密度 $\rho(\vec{x},t) = e\delta^3(\vec{x}-\vec{r}(t))$ … (1.3)

・電流密度 $\vec{i}(\vec{x},t) = e\dot{\vec{r}}(t)\delta^3(\vec{x}-\vec{r}(t))$ … (1.4)

$$(\delta^3(\vec{x}-\vec{r}(t)) = \delta(x-x(t))\,\delta(y-y(t))\,\delta(z-z(t)))$$

(1.3) と (1.4) は電荷保存則 (第1章 (6.1)) をみたすので,

$$\frac{\partial\rho(\vec{x},t)}{\partial t} + \mathrm{div}\,\vec{i}(\vec{x},t) = e\left[\frac{\partial}{\partial t}\delta^3(\vec{x}-\vec{r}(t)) + \dot{\vec{r}}(t)\,\mathrm{grad}_{\vec{x}}\,\delta^3(\vec{x}-\vec{r}(t))\right]$$

$$= e\left[\mathrm{grad}_{\vec{r}}\,\delta^3(\vec{x}-\vec{r}(t))\,\dot{\vec{r}}(t) - \dot{\vec{r}}\,\mathrm{grad}_{\vec{r}}\,\delta^3(\vec{x}-\vec{r}(t))\right]$$

$$= 0$$

$$\left(\begin{aligned} \mathrm{div}\,\vec{A} &= \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} \\ \mathrm{grad} &= \vec{\nabla} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) \end{aligned}\right)$$

点電荷に作用する力 … 自己力をもふくむ Lorents の力 (第1章 (7.14))

$$\vec{F}(t) = \int d^3x\, \vec{f}(\vec{x},t)$$

$$= \int d^3x \left[e\delta^3(\vec{x}-\vec{r}(t))\,\vec{E}(\vec{x},t) + e\delta^3(\vec{x}-\vec{r}(t))\,\dot{\vec{r}}\times\vec{B}(\vec{x},t)\right]$$

$$= e\vec{E}(\vec{r}(t),t) + e\dot{\vec{r}}(t)\times\vec{B}(\vec{r}(t),t)$$

N個の質量 $m_i$, 電荷 $e_i$ の点電荷とそれにともなう電磁場がみたす方程式

$$m_i\frac{d^2\vec{r}(t)}{dt^2} = e_i\vec{E}(\vec{r}_i(t),t) + e_i\dot{\vec{r}}_i(t)\times\vec{B}(\vec{r}_i(t),t) \quad (i=1,2,\cdots N) \quad \cdots (1.6)$$

$$\mathrm{rot}\,\vec{H}(\vec{x},t) - \frac{\partial\vec{D}(\vec{x},t)}{\partial t} = \sum_{i=1}^{N} e_i\dot{\vec{r}}_i(t)\,\delta^3(\vec{x}-\vec{r}_i(t))$$

$$\mathrm{div}\,\vec{D}(\vec{x},t) = \sum_{i=1}^{N} e_i\,\delta^3(\vec{x}-\vec{r}_i(t))$$

$$\mathrm{div}\,B(\vec{x},t) = 0$$